

NWS OPERATIONAL DYNAMIC WAVE MODEL

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INTRODUCTION

The National Weather Service (NWS) hydrology program is to provide accurate and timely hydrologic information to the general public. This includes flood forecasts, as well as day-to-day river forecasts which are used for water supply, navigation, irrigation, power, reservoir operation, recreation, and water quality interests. Twelve River Forecast Centers prepare the forecasts which are disseminated to the public throughout the United States via local Weather Service Forecast Offices.

In the late 1960's, NWS began moving from an index type catchment runoff model to a continuous conceptual hydrologic model with a strong physical basis. The new conceptual model is now being implemented throughout the United States.

Where runoff generated from precipitation input to the conceptual model aggregates in fairly large, well-defined channels (rivers), it is transmitted downstream by routing techniques of the hydrologic or storage routing variety, e.g., the lag and K technique. Although the hydrologic routing techniques function adequately in many situations, they have serious shortcomings when the unsteady flows are subjected to backwater effects due to reservoirs, tides, or inflows from large tributaries. When channel bottom slopes are quite mild, the flow inertial effects ignored in the hydrologic technique also become important.

In the early 1970's, the NWS Hydrologic Research Laboratory began developing a dynamic wave routing model based on an implicit finite difference solution of the complete one-dimensional St. Venant equations of unsteady flow. This hydrodynamic model, known as DWOPER (Dynamic Wave Operational Model), has recently begun to be implemented where backwater effects and mild bottom slopes are most troublesome for hydrologic routing methods. It is either in operational service or in the process of being implemented on the Mississippi, Ohio, Columbia, Missouri, Arkansas, Red, Atchafalaya, Cumberland, Tennessee, Willamette, Platte, Kansas, Verdigris, Ouachita, and Yazoo Rivers.

DWOPER features the ability to use large time steps for slowly varying floods and to use cross-sections spaced at irregular intervals

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along the river system. The model is generalized for wide applicability to rivers of varying physical features, such as irregular geometry, variable roughness parameters, lateral inflows, flow diversions, off-channel storage, local head losses such as bridge contraction-expansions, lock and dam operations, and wind effects. It is suited for efficient application to dendritic river systems or to channel networks consisting of bifurcations with weir-type flow into the bifurcated channel. DWOPER has a highly efficient automatic calibration feature for determining the optimum roughness coefficients for either a single channel or system of interacting channels. Extensive data management programming features allow the model to be used in a day-to-day operational forecasting environment with minimal card coding required. It is also equally applicable for simulating unsteady flows for purposes of engineering planning, design, or analysis.

MODEL DESCRIPTION

Mathematical Basis - The basis for DWOPER is a finite difference solution of the conservation form of the one-dimensional equations of unsteady flow consisting of conservation of mass and momentum equations, i.e.,

$$\frac{\partial Q}{\partial x} + \frac{\partial(A+A_o)}{\partial t} - q = 0 \quad (1)$$

$$\frac{\partial Q}{\partial t} + \frac{\partial(Q^2/A)}{\partial x} + gA \left(\frac{\partial h}{\partial x} + S_f \right) - qv_x + W_f B = 0 \quad (2)$$

in which Q is discharge, A is cross-sectional area, A_o is off-channel cross-sectional area wherein flow velocity is considered negligible, q is lateral inflow or outflow, x is distance along the channel, t is time, g is gravity acceleration constant, v_x is the velocity of lateral inflow in the x -direction, W_f is the wind term, B is the channel top width and S_f is the friction slope defined as:

$$S_f = \frac{n^2 |Q| Q}{2.2 A^2 R^{4/3}} \quad (3)$$

in which n is the Manning roughness coefficient and R is the hydraulic radius.

Equations (1) and (2) are nonlinear partial differential equations which may be solved without necessary simplifications by finite difference techniques of explicit or implicit variety. Explicit methods, although simpler in application are not suitable for application of the equations to long-term unsteady flow phenomena such as flood waves because they are restricted by mathematical stability considerations to very small computational time steps (on the order of a few minutes); this causes the explicit techniques to be very inefficient in the use of computer time. Implicit finite difference techniques, however, have no restrictions on time step size other than accuracy considerations.

The "weighted four-point" implicit scheme is used since it appears to be the most advantageous of the various implicit schemes which have been proposed from time to time because it can readily be used with unequal distance steps and its stability-convergence properties can be controlled.

Using the weighted four-point implicit finite difference scheme to approximate the various terms in Equations (1) through (3) results in a system of $2N-2$ nonlinear algebraic equations, (N represents the number of cross-sections used to describe the routing reach). This system of equations has $2N$ unknowns, i.e., the discharge (Q) and the water surface elevation (h) at each cross-section. Two additional equations which represent prescribed boundary conditions at the upstream and downstream extremities of the routing reach are utilized to form a system of $2N$ equations with $2N$ unknowns which is determinate. The system of nonlinear equations is solved by the Newton-Raphson functional iterative technique.

Computations for the iterative solution of the nonlinear system are begun by assigning trial values to the $2N$ unknowns. Substitution of the trial values into the system of nonlinear equations yields a set of $2N$ residuals. The Newton-Raphson method provides a means for correcting the trial values until the residuals vanish or are reduced to tolerable magnitudes. A system of $2N \times 2N$ linear equations relate the corrections to the residuals and to a coefficient matrix composed of partial derivatives of each equation with respect to each unknown variable in that equation. The coefficient matrix of the linear system has a banded structure which allows the system to be solved by a compact quad-diagonal Gaussian elimination algorithm which is very efficient with respect to computing time and storage. The efficiency of the method is quite dependent on the success with which the first trial values are made. A method of parabolic extrapolation has been found to provide trial values sufficiently close to the unknowns to assure convergence within one to three iterations.

Initial Conditions - DWOPER allows initial conditions to be obtained from the following sources: 1) Estimated stages and discharges at each cross-section are read in; 2) Observed stages at each cross-section where a river gage is located are read in; stages at intermediate cross-sections are linearly interpolated within the model; observed discharge at the upstream extremity of the main stem river and each tributary are also read in; all downstream discharges are determined by summation of flows from the upstream to downstream boundaries; and 3) Computed stages and discharges which have been saved from a previous unsteady flow simulation.

Boundary Conditions - DWOPER can readily accommodate either of the following boundary conditions at the upstream extremities of the river system: 1) known stage (water surface elevation) hydrograph, $h_1(t)$; or 2) known discharge hydrograph, $Q_1(t)$. Downstream boundary conditions included as options in DWOPER are: 1) known stage hydrograph, $h_N(t)$; 2) known discharge hydrograph, $Q_N(t)$; or 3) a known relationship between stage and discharge such as a rating curve (single-valued or looped).

Cross-Sections - Cross-sections of irregular as well as regular geo-

metrical shape are acceptable in DWOPER. Each cross-section is read in as tabular values of channel width and elevation, which together constitute a piece-wise linear relationship. During the solution of the unsteady flow equations, any areas or widths associated with a particular water surface elevation are linearly interpolated from the piece-wise linear relationships of width to elevation read in or area elevation sets initially generated within the model.

Cross-sections at gaging station locations are always used as computational points in the x-t plane. Cross-sections are also specified at points along the river where significant cross-sectional changes occur or at points where major tributaries enter. Usually, "average" cross-sections are placed mid-way between cross-sections associated with gaging stations or significant geometry changes. The average cross-section is a weighted average of the cross-sectional properties of the intervening reach.

Off-Channel Storage - Dead storage areas wherein the flow velocity in the x-direction is considered negligible relative to the velocity in the active area of the cross-section is a feature of DWOPER. Such dead or off-channel storage areas can be used to effectively account for embayments, ravines, heavily wooded flood plains, or tributaries which connect to the flow channel but do not pass flow and serve only to store the flow. The off-channel storage cross-sectional properties are described in the same way as the active cross-sectional areas, i.e., for each section, a table of top widths and elevations is read in.

Roughness Coefficients - Manning's n is used to describe the resistance to flow due to channel roughness caused by bed forms, bank vegetation and obstructions, bend effects, and eddy losses. The Manning n is defined for each channel reach bounded by gaging stations and is specified as a function of either stage or discharge according to a piece-wise linear relation with both n and the independent variable (h or Q) read in to DWOPER in tabular form. Linear interpolation is used to obtain n for values of h or Q intermediate to the tabular values.

Lateral Inflows - DWOPER incorporates tributary inflows via the lateral inflow term, q , in Eqs. (1) and (2). The inflows are considered to be independent of flows occurring in the river to which they are added. They are read in as a time series of flows with constant or variable time intervals.

Lock and Dam Condition - A river system may include small dams with gates to pass the river flow in such a way as to maintain certain water surface elevations on the upstream side of the dam. Usually associated with the dam is a lock for allowing navigation of river craft and barges past the dam. DWOPER can accommodate any number of lock and dam installations within the river system being simulated. A "through" computation scheme is used as opposed to separating the river system into discrete portions because of the lock and dam and specifying external boundary conditions applicable to the lock and dam. The through computation scheme allows the simultaneous simulation of the entire river system including portions with lock and dams. This facilitates data preparation and allows a correct simulation of backwater effects when

the tailwater elevation below the dam raises to an elevation such that the pool elevation on the upstream side is no longer controlled by operation of the gates.

Dendritic River Systems - Although the implicit formulation of the unsteady flow equations is well suited for simulating unsteady flows in a system of rivers - in that the response of the system as a whole is determined during each time step - particular care must be given to maintain the necessary solution efficiency of the matrix solution technique of the Newton-Raphson procedure. An efficient solution technique for dendritic (tree-type) river systems is utilized in DWOPER. This technique solves during a time step the unsteady flow equations first for the main stem and then for each tributary of the river system. The tributary flow at the confluence of the tributary and main-stem river is treated as lateral flow q which is first estimated when solving the equations for the main stem. DWOPER can accommodate any number of tributaries. Although the iterative algorithm is designed for 1st order tributaries, systems with 2nd order tributaries may sometimes be accommodated by reordering the system i.e., selecting another branch of the system as the main stem.

Weir-Flow Bifurcations - In DWOPER, any number of Δx reaches along a channel may bypass flow to another channel which connects back into the former channel at some point downstream from the bifurcation. The flow in the bypass channel which may affect the weir flow is accounted for by a submergence correction to the weir flow. The crest elevation of the overbank section which acts as the weir-flow bypass is specified. Each section has a discharge coefficient which may be estimated or obtained through trial-error calibration. The location along the channel where the bifurcation(s) occur, the average crest elevation of each such Δx reach, and the discharge coefficient are read in as input data.

Automatic Calibration - A critical task in the application of one-dimensional hydrodynamic models to natural rivers in dendritic systems is the determination of the roughness parameter in the friction slope term of the momentum equation. The roughness parameter often varies with discharge or stage and with distance along the river. The DWOPER model contains an internal feature which can be accessed to automatically determine the optimum roughness parameter which will minimize the difference between computed and observed values. The automatic calibration feature is a simple and highly efficient optimization technique for determining the continuous piece-wise linear variation of the roughness parameter with discharge (or stage) for each reach of the river bounded by gaging stations. The optimization technique is based on a decomposition principle which simplifies the treatment of complex river systems of dendritic configuration.

Data Management Module - Preparation of the data for simulation of a river system requires a substantial amount of work. The river system configuration, cross-sections, etc. must be determined and coded. Stage and discharge data for the boundary and initial conditions must be determined and card coded. This initial work cannot be avoided; however, the data management module does substantially reduce the time and effort required to use the model on a day-to-day operational basis. The data

initially card coded to simulate a particular river system are kept on disk, a direct access peripheral mass storage, and only the updated information for boundary conditions need be card coded and read in before a new simulation can be made.

The data stored on disk are of two types: stationary data which does not change with time and stage-discharge data which must be updated as new observations are reported. The stationary data are stored in "carryover" files and stage-discharge data are stored in "hydrograph" files. In order to perform a forecasting run, the river system configuration and physical properties must be determined by retrieving the data in a carryover file and the stage-discharge data must be retrieved from a hydrograph file. After the initial simulation run, the initial conditions which consist of the stages and discharges at every computational point in the river system are available in the carryover file as computed stages and discharges which have been stored from a previous run.

The data management module element and the dynamic wave computational element are accessed by commands. Each command causes the program to branch to appropriate subroutines where the desired function is performed. Some of the more significant data management commands are:

- 1) COINIT - The Carryover file is initialized, i.e., the river configuration, cross-section properties, etc., are read in.
- 2) HINIT - The Hydrograph file is initialized, i.e., the hydrograph values are read in.
- 3) COEDIT - Any stationary data contained in the carryover file may be updated by simple reference indicators punched on input cards along with the updated value(s).
- 4) HEDIT - Any data contained in the hydrograph file may be deleted, replaced, or added to by simple label and time period indicators punched on input cards along with the new hydrograph values.
- 5) COLIST - List the contents of a particular carryover file.
- 6) HLIST - List the contents of a particular hydrograph file.

Some of the commands used to activate the dynamic wave computational element are:

- 1) RUN - Simulates a river system using data from carryover and hydrograph files stored on disk.
- 2) ICSAVE - A command used concurrently with RUN command. ICSAVE is used to save the water surface elevations and discharges at all computational points at a specified time. These values are retained in peripheral storage for use in subsequent simulation runs as the appropriate and necessary initial conditions.

- 3) ALONE - Simulates a river system using data (river system configuration, cross-section properties, hydrograph values at boundaries, etc.) read in on cards at the same time as the ALONE command is read in.

Computer Core and Computational Requirements - DWOPER has been created using the programming feature, "variable dimensioning". This enables the size of the arrays of subscripted variables such as observed hydrograph stages, cross-section top widths, computed stages and discharges, etc., to be changed from one simulation run to the next. There is a maximum total size for the sum of all arrays, but within that bound the allocation of core space among the variables is flexible and is specified as a data input which is retained in the carryover file. Variable dimensioning, together with another programming feature known as "over-laying", in which groups of subroutines are loaded into core as needed, reduce the required core to the range of 170 to 250 K words.

The implicit formulation of the basic dynamic wave computational element allows the time step size to be selected according to accuracy requirements rather than numerical stability considerations. This factor makes DWOPER very efficient in the use of computer time. Efficiency due to the implicit formulation compared to computational requirements of explicit finite difference models is greatest for slowly varying transients in large rivers and decreases as the transient being modeled becomes more rapidly varying. Computational requirements are approximately 0.004 sec/time step/distance step. Some examples of total computational time (IBM 360/195) are: 1) Mississippi-Ohio-Cumberland-Tennessee River system: consisting of 393 miles of river, 35 computational points, 3 months of simulation time, 20 observed and computed hydrographs; slowly varying flood wave simulation using 24 hour time steps required 15 seconds of CPU time; and 2) Lower Columbia and Willamette River systems: consisting of 155 miles of river, 25 computational points, 9 days of simulation time, 10 observed and computed hydrographs; tidal flows with 12 hour period and 1 hour time steps required 19 seconds of CPU time.

SELECTED APPLICATION

Mississippi-Ohio-Cumberland-Tennessee System - A dendritic river system consisting of 393 miles of the Mississippi-Ohio-Cumberland-Tennessee (MOCT) River system was simulated using DWOPER. A schematic of the river system with 16 gaging stations is shown in Fig. 1. In applying DWOPER to this system, the main-stem river is considered to be the Ohio-Lower Mississippi segment with the Cumberland, Tennessee, and Upper Mississippi considered as first-order tributaries. The channel bottom slope is mild, varying from about 0.25 to 0.50 ft/mi. Each branch of the river system is influenced by backwater from downstream branches. Total discharge through the system varies from low flows of approximately 120,000 cfs to flood flows of 1,700,000 cfs. A total of 45 cross-sections located at unequal intervals ranging from 0.5 - 21 miles were used to describe the MOCT river system.

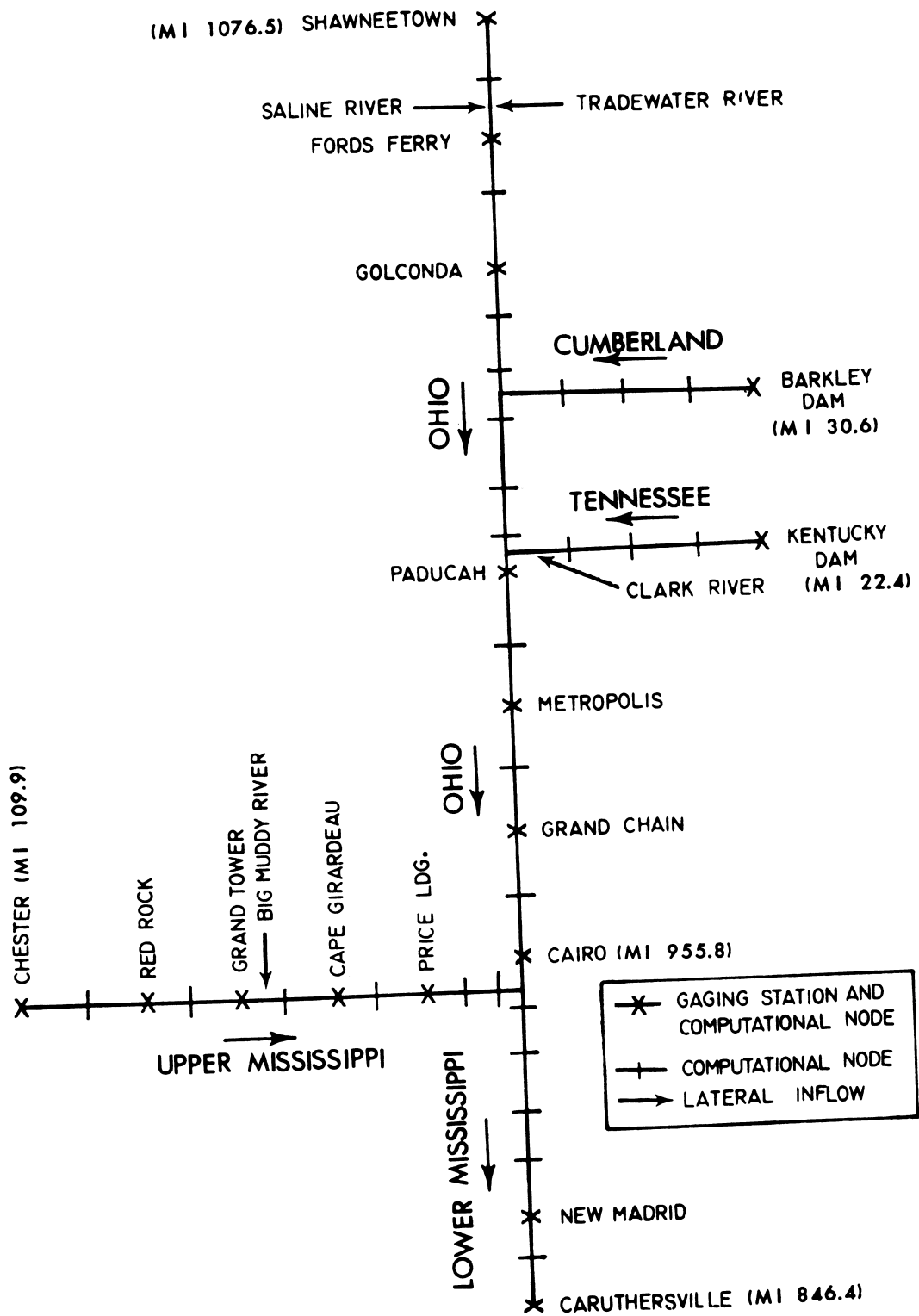


FIG. 1 - SCHEMATIC OF MISSISSIPPI-OHIO-CUMBERLAND-TENNESSEE (MOCT) RIVER SYSTEM

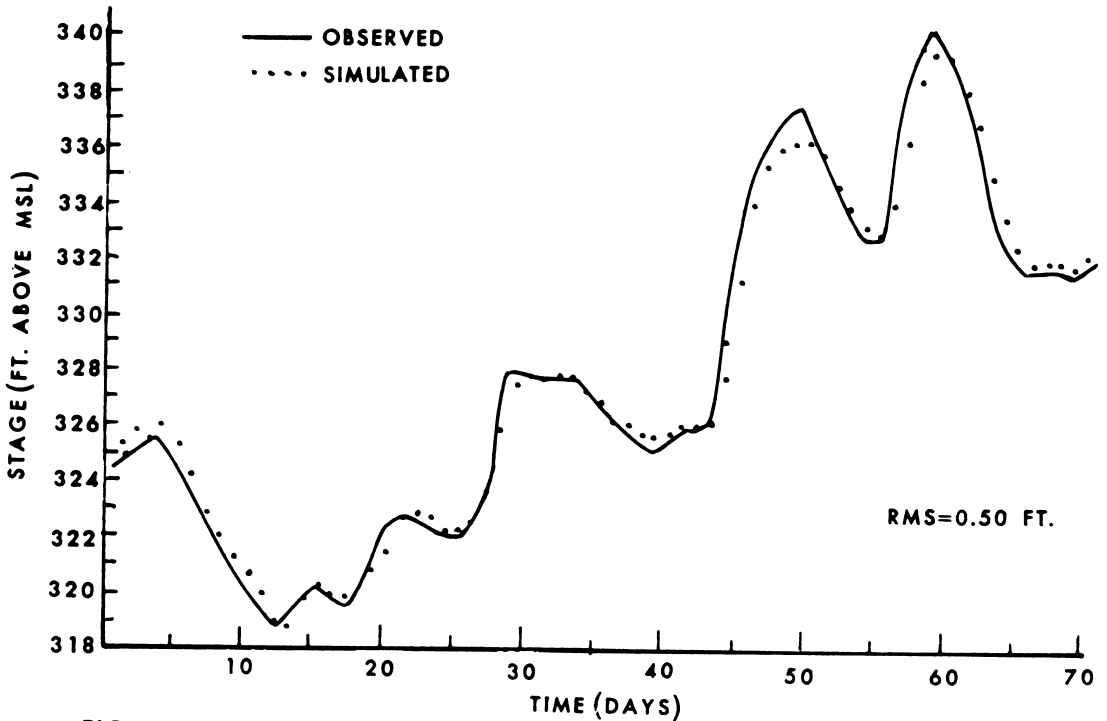


FIG. 2 A - OBSERVED VS. SIMULATED STAGES AT CAPE GIRARDEAU FOR 1970 FLOOD

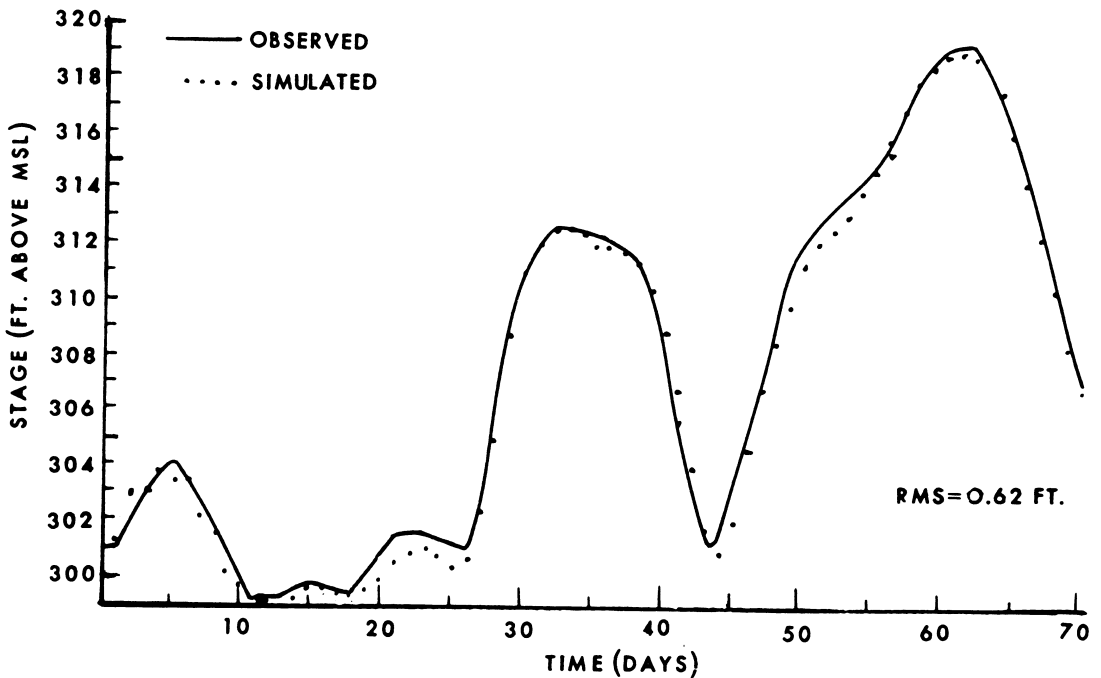


FIG. 2 B - OBSERVED VS. SIMULATED STAGES AT CAIRO FOR 1970 FLOOD

The MOCT system was calibrated to determine the n - Q relationship for each of 15 reaches bounded by gaging stations. Time steps of 24 hours were used. About 25 seconds of IBM 360/195 CPU time were required by DWOPER to perform the calibration. The flood of 1970 was used in the automatic calibration process. The average RMS error for all 15 reaches was 0.60 feet. A typical comparison of observed and simulated stages for the Cairo and Cape Girardeau gaging stations is shown in Fig. 3. Using the calibrated n - Q relations, the 1969 flood was simulated with DWOPER. Stage hydrographs at Shawneetown and Chester and discharge hydrographs at Barkley Dam and Kentucky Dam were used as upstream boundary conditions, and a rating curve was used as the downstream boundary condition at Caruthersville. The average RMS error for the 11 intermediate gaging stations was 0.56 feet.

SUMMARY

An operational hydrodynamic model (DWOPER) developed by the Hydrologic Research Laboratory of the National Weather Service is being placed in operational use by River Forecast Centers on a number of major river systems where storage routing methods are inadequate due to the effects of backwater, tides, and mild channel bottom slopes. The model is based on the complete one-dimensional St. Venant equations and belongs to the category of dynamic wave flood routing models. A weighted four-point nonlinear implicit finite difference scheme is used to obtain solutions to the St. Venant equations via a Newton-Raphson iterative technique. DWOPER has a number of features which make it applicable to a variety of natural river systems for real-time forecasting. It is designed to accommodate various boundary conditions and irregular cross-sections located at unequal distances along a single multiple-reach river or several such rivers having a dendritic configuration. It allows for roughness parameters to vary with location and stage or discharge. Temporally varying lateral inflows, wind effects, bridge effects, off-channel storage, weir-flow channel bifurcations are included among its features. Time steps are chosen solely on the basis of desired accuracy since the implicit finite difference technique is not restricted to the very small time steps of explicit techniques due to numerical stability considerations. This enables DWOPER to be very efficient as to computational time for simulating slowly varying floods of several days duration. An efficient automatic calibration procedure for determining optimum Manning n - stage or discharge relationships from observed data is provided as an option in DWOPER. Data handling requirements for day-to-day river forecasting are minimal due to extensive data management features utilizing disk or tape storage. Operationally, card coding is only required to update hydrograph files with the most recent observations. Applications of DWOPER to several large river systems have demonstrated its operational efficiency, accuracy, and utility. The model is currently being extended to account for effects of channel sinuosity, flood plains, sediment transport, and bank storage on unsteady flows in alluvial rivers.